THE
DESCRIPTION AND USE
OF A
CASE
OF
Mathematical Inſtruments;

PARTICULARLY OF ALL THE
LINES contained on the PLAIN SCALE, the
SECTOR, the GUNTER, and the *Proportional*
COMPASSES.

WITH A
PRACTICAL APPLICATION,
Exemplified in many uſeful Caſes of GEOMETRY, and
Plain and *Spherical* TRIGONOMETRY.

The Whole illuſtrated by
COPPER-PLATE FIGURES.

K

By BENJAMIN MARTIN.

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BRITISH MUSEUM

THE
DESCRIPTION AND USE
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CASE
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Mathematical Inſtruments.

A POCKET CASE of MATHEMATICAL INSTRUMENTS contains the following Particulars, viz. (1.) A PAIR of *Plain* COMPASSES. (2.) A PAIR of *Drawing* COMPASSES, with its ſeveral Parts. (3.) A *Drawing* PEN and *Pointer*. (4.) A PROTRACTOR, in Form of a *Semicircle*, or, ſometimes, of a *Parallelogram*. (5.) A PARALLEL RULER. (6.) A *Plain* SCALE. (7.) A SECTOR: Beſides the *Black-Lead Pencil* for drawing Lines. The general Uſes of the above Inſtruments are as follow:

I. *Of the* PLAIN COMPASSES. Fig. 1.

THE Uſe of the Common or *Plain Compaſſes* is (1.) to draw a *blank Line* A B, by the Edge of a Ruler, through any given Point or Points C D, &c. (2.) Take any Extent or Length between the Points of the Compaſſes, and to ſet it off, or apply it ſucceſſively upon any Line, as from C to D. (Fig. 2.) (3.) To take any propoſed Line CD between the Points, and, by applying it to the proper

Scale, to find its Length. (4.) To ſet off *Equal Diſtances* upon a given Line, by making a Dot with the Point at each, through which to draw parallel Lines. (5.) To draw any blank *Circle*, interſecting *Arches*, &c. (6.) To lay off an Angle of a given Quantity upon an Arch of a Circle from the Line of Chords, &c. (7.) To meaſure any Arch, or Angle, upon the Chords, &c. (8.) To conſtruct any propoſed Figure, in plotting or making Plans, &c. by ſetting off the Quantity of the Sides and Angles from proper Scales. In ſhort, the Uſe of the Compaſſes occurs in every Branch of practical Mathematics, as we ſhall ſee more particularly hereafter.

II. *Of the* DRAWING COMPASSES.

Theſe Compaſſes are chiefly deſigned for drawing Circles, and circular Arches; and it is often neceſſary they ſhould be drawn with different Materials; and therefore this Pair of Compaſſes has, in one of its Legs, a triangular *Socket*, and *Screw*, to receive and faſten the following Parts or Points for that Purpoſe, viz. (1.) A *Steel Point*, which being fixed in the Socket, makes the Compaſſes then but a *Plain Pair*, and has all the ſame Uſes as juſt now deſcribed in drawing Blank Circles, ſetting off Lines, &c. (2.) A *Port Crayon* with a *Black-lead Pencil*, cut to a fine Point, for drawing Lines that may be eaſily rubb'd out again, if not right. A Piece of Slate Pencil may alſo be uſed in this Part for drawing on *Slate*. (3.) The *Dotting Point*, or *Dotting Pen* with a ſmall *Rowel*, or indented Wheel at the End, moving very freely; and receiving Ink from the Braſs Pen over it, communicates the ſame in equal and regular *Dots* upon the Paper, where dotted Lines are choſen. (4.) The *Steel Pen* or *Point*, for drawing and deſcribing *Black Lines* with *Ink*; for this Purpoſe the two Parts or Sides of the Pen are opened or cloſed, with an adjuſting Screw, that the Line drawn may be as fine or as coarſe as you pleaſe.

in the *Port-Crayon*, *Dotter*, and *Steel-Pen* there is a Joint, by which you can ſet the lower Part always *perpendicular* to the Paper, which is neceſſary for drawing a Line well, in every Extent or opening of the Compaſſes.

In ſome of the better Sort of Inſtruments, theſe Points ſlide into the Socket, and are kept tight by a Spring on the Inſide that is not ſeen.

The *Steel-Point* is ſometimes made with a Joint, and furniſhed with a *fine Spring and Screw*; by which, when you have opened the Compaſſes nearly to the Extent required, you can, by turning the Screw, move the Point to the true Extent, as it were, to a *Hair's Breadth*; which is the Reaſon theſe are called *Hair Compaſſes.*

The common Compaſſes, at large, are not altogether ſo well adapted for *ſmall Drawings*; and therefore a ſmall Sort, called *Bowes*, are contrived to anſwer all ſuch Purpoſes; they conſiſt only of a *Steel-Point* and *Drawing Pen*, with a Joint, and of a ſmall Length, ſo that very ſmall Circles may be nicely drawn with them, as they are to be conveniently moved and turned about in the Hand, by a ſhort Stem or Shaft.

III. *Of the* DRAWING PEN *and* PENCIL.

The *Drawing Pen* is only the common Steel Pen at the End of a Braſs Rod, or Shaft, of a convenient Length, to be held in the Hand for drawing all Kinds of *ſtrait* Black Lines by the Edge of a Rule. The Shaft or Handle has a Screw in the middle Part; and, when unſcrewed, there is a *fine round Steel Pin, or Point*, by which you make as nice a Mark or Dot upon the Paper as you pleaſe, for terminating your Lines in curious Draughts.

The *Black-Lead Pencil*, if good, is of frequent Uſe for drawing ſtrait Lines; and for ſupplying the Place of the

Drawing Pen, where Lines of Ink are not neceſſary; it is alſo often ſubſtituted for the *Common Pen* in *Writing*, *Figuring*, &c. Becauſe in all Caſes, if what be drawn with it be not right, or does not pleaſe, it may be very eaſy rubb'd out with a Piece of Crum-Bread, and the whole new-drawn.

IV. *Of the* PROTRACTOR.

The *Protractor* is a Semicircle of Braſs, divided into 180 Degrees, and numbered each Way from End to End of the Semicircle by 10°, 20°, 30°, &c. The *Central Line* is the external Edge of the Protractor's Diameter, or ſtrait Side; ſloped down to the under Side, and is generally called a *Fiducial Edge*; in the Middle of which is a ſmall Line or fine Notch in the very Edge, for the *Center* of the Protractor. The Uſes of the Protractor are Two, (1.) To meaſure any Angle propoſed. (2.) To lay down any Angle required.

For Example, Suppoſe it required to find what Number of Degrees are contained in the Angle ACB (Fig. 3.) you place the Center of the Protractor upon the Angular Point C, and the Fiducial Edge exactly upon the Line C A; then obſerve what Number of Degrees the Line C B cuts upon the graduated Limb of the Protractor, and that will be the Meaſure of the Angle A C B, as required.

Secondly, Suppoſe it required to protract or lay off from the Line A C, an Angle A C B, equal 35 Degrees. To do this, you place the Center of the Protractor upon the given Point C, and the ſtrait Edge upon A C, very exactly; then make a fine Point or Dot at 35 Degrees on the Limb at B, and, the Protractor being removed, you draw through B, the ſtrait Line C B, and it will make the Angle A C B required.

Protractors in Form of a *Parallelogram*, or Long Square, are uſually made in Ivory or Braſs; are more exact than the common Semicircular ones, for Angles to 40 or 50 Degrees; becauſe at and about each End, the Diviſions (being farther from the Center) are larger; but the Advantage ſcarcely compenſates the Expence.

V. *Of the* PARALLEL RULER.

The *Parallel Ruler* is ſo called, becauſe as it conſiſts of two ſtrait Rules, connected together by two Braſs Bars, yet ſo as to admit a very free Motion to each; the one Ruler muſt always move *parallel-wiſe* to the other, that is, one Rule will be every where equi-diſtant from the other, and by this Means it becomes naturally fitted for drawing one or more Lines *parallel to*, or equally diſtant from, any Line propoſed. The Manner of doing which is thus:

Let it be required to draw a ſtrait Line, parallel to a given Line A B, and at the Diſtance A C from it. (Fig. 4) Firſt open the Rulers to a greater Diſtance than A C, and place the Edge of one of the Rules exactly on the Line A B; then holding the other Rule (or Side) firmly on the Paper, you move the upper Rule down from A to the Point C, by which (holding it faſt) you draw the Line C D, which will be parallel to the given Line A B, as required.

Many very uſeful Problems in the Mathematics are performed by this Inſtrument; of which the following are Examples.

Let it be required to find a *Fourth Proportional* to three Right Lines given, A B, B C, and A D. (Fig. 5.) To do this, draw the Lines A C, A E, making any Angle at Pleaſure. Upon A C with the Compaſſes ſet off the Lines A B and B C; and upon A E ſet off the Line A D; join

D B, and parallel to it draw E C, then will D E be the Fourth Proportional required. For A B : B C : : A D : D E.

Again, Suppoſe it required to divide any Line A B, as another Line A C is divided (Fig. 6.) To do this, join the Extremities of each Line C B, and parallel to C B draw E I, E H, D G through the given Points D E F in the Line A C; and by theſe Lines the Line A B will be divided exactly ſimilar to the Line A C.

The Parallel Ruler is ſeldom put into a Caſe of Inſtruments, but thoſe of the larger and better Sort; being generally ſold by itſelf of various Sizes, from 6 Inches to 2 Feet in Length.

VI. *Of the* PLAIN SCALE.

The *Plain Scale*, in common Caſes of Inſtruments, has the following Lines or Scales upon it, viz. (1.) A *Line* of 6 *Inches*. (2.) A *Line* of 50 *equal Parts*. (3.) A *Diagonal Scale*. Theſe are put on one Side. On the other Side are, (4.) A *Line* of *Chords*, marked C, (5.) Seven particular Scales of equal Parts, or *Decimal Scales*, of different Sizes; the Numbers placed at the Beginning of each denote how many of the ſmall Diviſions at the Beginning are contained in one Inch, viz. 10, 15, 20, 25, 30, 35, 40.

The Uſe of the *Line of Inches* is the ſame in this as in all other Rules, viz. to take the Length or Dimenſions of Bodies in *Inches*, and *Tenths* of an *Inch*, in order to compute their Contents.

The Line of 50 equal Parts, being equal to 6 Inches, ſhews the *Foot* to be divided into 100 of the ſame equal Parts, and the Diviſions of this Line are placed by thoſe of the Inches, that it may be eaſily ſeen what Number in one is equal to a given Number of the other; thus 3 Inches is equal to 25 Parts of the 100. And 30 of theſe latter are equal to 3 Inches and 6 Tenths. This Line is therefore often uſeful in practical Mathematics.

The *Diagonal Scale* is properly a *Centeſſimal Scale*, becauſe by it you divide an Unit into 100 equal Parts; and therefore you can lay off, or expreſs, any Number to the 100th Part of an Unit, which is an Exactneſs generally ſufficient in practical Buſineſs. How this is done will be eaſy to underſtand, as follows: Let A B be 1 or Unit, and divide it into 10 equal Parts at 1, 2, 3, 4, &c. At a proper Diſtance B C, draw the Line C D equal and parallel to A B, and divide this Line C D alſo into 10 equal Parts at *a*, *b*, *c*, *d*, &c. then join the Points A *a*, 1 *b*, 2 *c*, 3 *d*, &c. and theſe will be the 10 *Diagonal Lines*. Laſtly, divide B C into 10 equal Parts alſo, and number them 1, 2, 3, 4, &c. to 10 at C; then through each of theſe Diviſions draw Lines parallel to A B, through the Length of the Scale, and the Conſtruction is compleated. See Fig. 7.

In this Diagonal Scale (upon the Plate) A B is one Inch; then if it be required to take off $1\frac{73}{100}$ Inches, or 1,73; ſet one Foot of the Compaſſes in the 3d Parallel under 1 at *e*, and extend the other Foot or Point to the 7th Diagonal in that Parallel at *g*; and the Diſtance *e g* is that required; for *e f* is 1 Inch, and *f g* is 73 Parts of 100.

Again, ſuppoſe it required to ſet off upon any Line 2,37 Inches; then place one Point of the Compaſſes on the 7th Parallel under 2 at *h*, and extend the other to the 3d Diagonal in the ſame Parallel at *i*; and the Diſtance *h i* is that required. Or if A B be 10, the Diſtance *e g* is 17,3; and *h i* is 23,7. Alſo, if A B be 100, then *e g* is 173, *h i* is 237; and ſo on.

This Diagonal Scale has this Centeſſimal Diviſion at each End, and the Unit in one is juſt double of that at the other; thus if A B be 1 Inch at one End, it is ½ an Inch at the other; or if it be ½ an Inch in the larger, it is ¼ Inch in the leſſer Diviſions, as is the Caſe upon moſt of the common *Plain Scales*.

This Unit A B may alſo be 1 Foot, 1 Yard, 1 Rod, 1 Mile, &c. So that every Unit in every Kind of Meaſures is hereby eſtimated in 100th Parts of the Whole, which ſhews the *Diagonal Scale* to be a moſt uſeful Invention.

On the other Side of the Plain Scale are the Seven Decimal Lines, which are uſually called *Plotting Scales*, becauſe their Diviſions of an Unit into 10 Parts being different in Proportion of 4 to 1, the Surveyor has it in his Power to vary the Scale of his Plot or Plan of an Eſtate, &c. in that Ratio, in Seven different Drawings; and the Superficies or Sizes of the greateſt and leaſt Plans will be as 16 to 1. Or that drawn by Scale Nº 10, will be 16 Times larger than the Plan laid down from Scale Nº 40.

Alſo the ſame Variety is to be had in the Conſtruction of all other Geometrical Figures, where *Superficies*, or *Solids*; and with reſpect to the latter, the greateſt will be to the leaſt as 64 to 1; that is, the *Architect* can vary the Size of his *Houſe*, *Temple*, &c. in the Ratio of 64 to 1 in 7 different Elevations.

The laſt *Line* on the common Plain Scale is that of *Chords*; and is much more uſed than the Protractor for laying off or meaſuring any propoſed Angle. Thus let it be required to draw the Line B C to make an Angle of 35 Degrees with the Line A C (Fig. 3.) to do this, ſet one Point of the Compaſſes in the Beginning of the Line of Chords, and extend the other to 60; with that Extent (as Radius) place one Foot in C, and with the other deſcribe the Arch A D; then take from the Chords 35° in the Compaſſes, and ſet them on the Arch from A to B; and through B draw C B, and it is done.

Again, Suppoſe it required to meaſure any Angle A B C propoſed, proceed thus; produce C A indefinitely, take

60° from the Chords in the Compaſſes, and with one Foot in C ſtrike the Arch A D, cutting the Leg B C in B; then take the Arch A B in the Compaſſes, and applying it upon the Beginning of the Line of Chords, it will reach to 35°, the Quantity of the Angle required. But the *Line of Chords* is more uſeful on the *Sector*, to which we now proceed.

Note. This Line of Chords is ſo conſtructed on my *Navigation Scales*, with Sexageſſimal Diviſions, that any Angle may be meaſured, or laid off, to a Minute of a Degree.

VII. *Of the* SECTOR.

The Sector is the moſt uſeful of all Mathematical Inſtruments; for it not only contains all the moſt uſeful Lines or Scales, but, by its Nature, renders them univerſal, as we ſhall ſhew in the following Deſcription:

The Lines commonly laid down upon the Sector are theſe (1.) A *Line of Equal Parts*, marked L at the End. (2.) A *Line of Chords* to 60 Degrees, marked C. (3.) A *Line of Sines*, 90 Degrees, marked S. (4.) A *Line of Tangents*, to 45°, marked T. (5.) Another Line of Tangents, from 45 to 75 or upwards, marked *t a*. (6.) A *Line of Secants*, marked *s e*. (7.) A *Line of Polygons*, marked POL.

Beſides theſe, when the Sector is quite opened, there is on one Side, (1.) *Gunter Line of Artificial Numbers*, *n*. (2.) *Line of Artificial Sines*, *s*. (3.) *Line of Artificial* Tangents, *t*. There is alſo a Line of 12 Inches, and another of the Foot divided into 1000 Equal Parts, placed by it, for the Purpoſes already mentioned.

Before a proper Idea can be formed of theſe Sectoral Lines, and their Uſes, we muſt ſhew their Conſtruction from the Circle. To this End, let A G B, (Fig. 8.) be a Quarter of a Circle divided into 90 Degrees, deſcribed with the Radius A C, on the Center C; let A F and C F be perpendicular to A C, in A and C; then if the Radius A C be divided into 10, 100, 1000, &c. *Equal Parts*, it will be the Line ſo called upon the Sector.

If from A, a Line be drawn to any Part or Diviſion of the Quadrant, as G at 60°, then that Line A G is the Chord of that Arch, or of 60°. And if the Line A B be drawn, it will be the Chord of 90°; and by ſetting one Foot of the Compaſſes in A, and extending the other to the ſeveral Diviſions 10, 20, 30, 40, &c. they may be transferred from the Circle to the Line A B, which will then be properly divided into a *Line of Chords* in 10, 20, 30, 40, &c. to 90, as on the Plane Scale.

If from any Point G in the Quadrant you let fall a Perpendicular G I to the Radius A C, or G H to the Radius C B, then the Line G I is called the *Sine* of the Arch A G; and the Line G H is the *Sine* of the Arch G B, the Complement of A G to 90°. And if all the Diviſions of the Quadrant were transferred to C B by Lines G H parallel to A B; then the Line C B will be divided as a *Line of Sines* in the Points 10, 20, 30, 40, &c. to 90°, as on the Sector you ſee it.

By laying a Rule from the Center C to the ſeveral Diviſions of the Quadrant 10, 20, 30, &c. it will cut the Line A E in the Points 10, 20, 30, &c. which will be thereby divided into a *Line of Tangents*; and here it muſt be obſerved that the Line of Tangents T on the Sector, extends but to 45° equal to A D (or B C) Radius. And that the Line of leſſer Tangents *t*, are projected from a leſſer Radius, and begin from 45° at the Diſtance of its Radius from the Center of the Sector.

For a CASE of Mathematical INSTRUMENTS

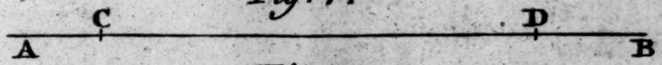
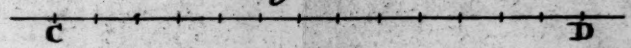


Fig. 1.



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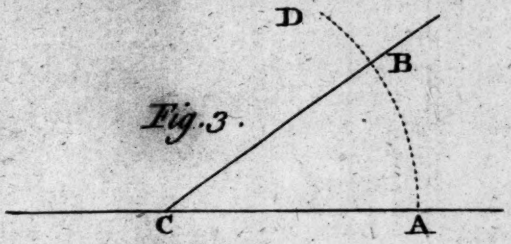

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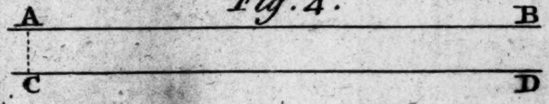

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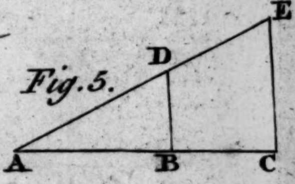

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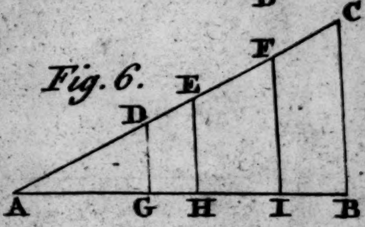

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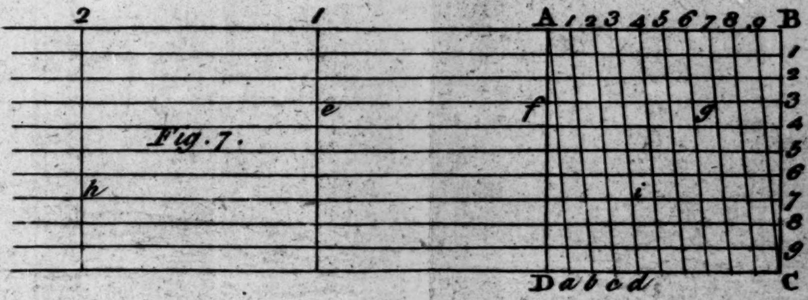


Fig. 7.

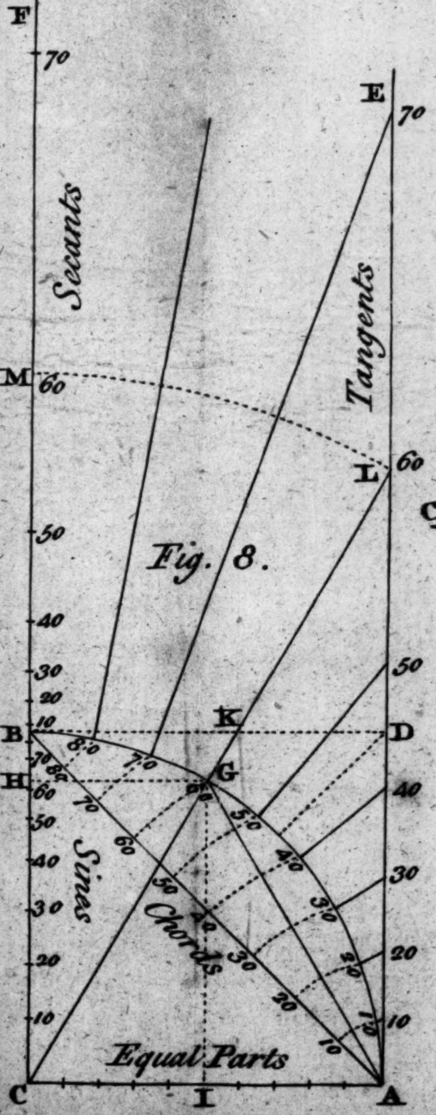


Fig. 8.

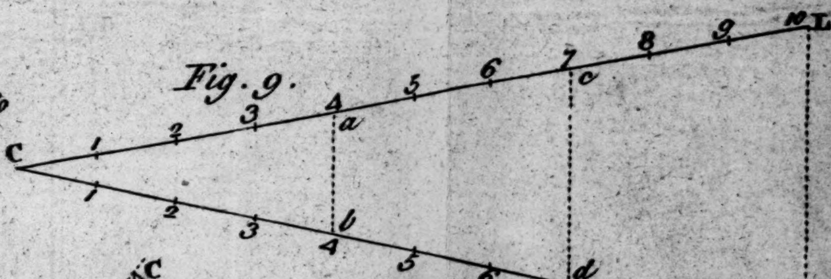


Fig. 9.

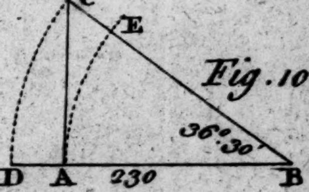
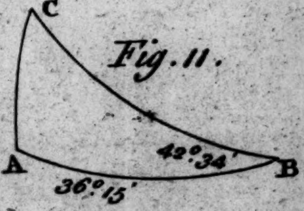


Fig. 10.



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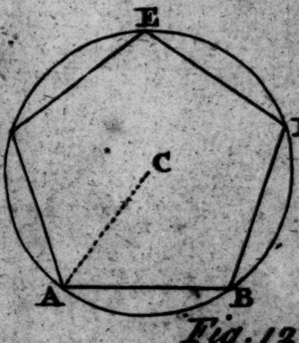

Fig. 12.

By drawing the Line C L through the Diviſion 60° to the Line A E, it makes A L, the *Tangent* of 60°, and is itſelf the *Secant* of 60°, or of the Angle A C L. And if with one Foot of the Compaſſes in C you extend the other to the ſeveral Diviſions in the Line A E, and transfer them to the Line C E, then will the Part B F be thus divided into a *Line of Secants*; being placed at the Diſtance of the Radius C B, from the Center of the Sector, and beginning at B, where the Radius ends.

It may be of uſe in many Caſes to obſerve, (1.) That the *Chord* of 60° A G, is equal to Radius A C or CG: (2.) That the *Sine* of 60° G I, biſects the Radius A C in I; and therefore the Sine GH of 30° is equal to ½ the Radius or C I. (3.) And therefore the Secant CL of 60°. is equal to twice the Radius A C; for C I is to C G as CA to CL. (4.) Therefore *Coſine is to Radius* as *Radius to the Secant.* (5.) Alſo the Tangent AL is to Radius AC as Radius B C is to the Co-tangent B K.

From what has been ſaid, the Reaſon appears why the *Line of Lines* (or equal Parts L) terminates upon the Sector at 10; the *Line of Chords* C at 60°; the *Line of Sines* S at 90°; the larger Tangents T at 45°; and that the *leſſer Tangents*, and alſo the *Secants*, are of indefinite Length.

From the Nature of the Sector, conſiſting of two *Pairs*, or Legs, moveable upon a central Joint, it is requiſite that that the Lines ſhould be laid on the Sector by Pairs, *viz.* one of a Sort on each Leg, and all of them iſſuing from the Center, all of the ſame Length, and every two containing the ſame Angle. We ſhall now illuſtrate the Nature of working Problems *Sectoral-wiſe*, as follows, by the *Lines of Lines*, or *Equal Parts*, L L.

Let CL, CL. (Fig. 9.) be the *two Lines of Lines* upon the Sector, opened to an Angle L C L; join the

Diviſions 4 and 4, 7 and 7, 10 and 10, by the dotted Lines *a b*, *c d*, L L. Then by the Nature of ſimilar Triangles, it is C L to C *b*, as L L to *a b*; and C L to C *d*, as L L to *c d*; and therefore *a b* is the ſame Part of L L as C *b* is of C L. Conſequently, if L L be 10, then *a b* will be 4, and *c d* will be 7 of the ſame Parts.

And hence, though the *Lateral Scale* C L be fixed, yet a *Parallel Scale* L L, is obtainable at Pleaſure; and therefore though the *lateral Radius* is of a determined Length in the Lines of *Chords*, *Sines*, *Tangents* and *Secants*, yet the *Parallel Radius* may be had of any Size you want, by means of the Sector, as far as its Length will admit; and all the *Parallel Sines*, &c. peculiar to it; as will be evident by the following Examples in each Pair of Lines.

EXAMPLE I. In the LINES OF EQUAL PARTS.

Having 3 Numbers given, 4, 7, 16, to find out a 4th Proportional. To do this, take the *lateral Extent* of 16 in the Line C L, and apply it *Parallel-wiſe*, from 4 to 4, by a proper Opening of the Sector; then take the parallel Diſtance from 7 to 7 in your Compaſſes, and applying one Foot in C, the other will fall on 28 in the Line of Lines C L, and is the Number required; for 4 : 7 :: 16 : 28.

EXAMPLE II. In the LINES OF CHORDS.

Suppoſe it required to lay off an Angle A C B, (Fig. 3.) equal to 35°; then with any convenient Opening of the Sector, take the Extent from 60 to 60, and with it (as Radius) on the Point C deſcribe the Arch A D indefifinitely; then in the ſame Opening of the Sector take the parallel Diſtance from 35° to 35°, and ſet it from A to B. in the Arch A D, and draw A B, and it makes the Angle at C required.

EXAMPLE III. In the LINES OF SINES.

The Lines of *Sines*, *Tangents*, and *Secants*, are uſed in Conjunction with the *Lines of Lines*, in the Solution of all the Caſes of Plain Trigonometry; thus, let there be given in the Triangle A B C, (Fig. 10.) the Side A B =230; and the Angle A B C=36° 30′; to find the Side A C. Here the Angle at C is 53° 30′. Then take the lateral Diſtance 230, from the *Line of Lines*, and make it a Parallel from 53° 30′ to 53° 30′ in the *Line of Sines*; then the parallel Diſtance between 36° 30′ in the ſame Lines, will reach laterally from the Center to 170, 19 in the Line of Lines for the Side A C required.

EXAMPLE IV. In the LINES OF TANGENTS.

If inſtead of making the Side B C Radius (as before) you make A B Radius; then A C, which before was a *Sine*, is now the Tangent of the Angle B; and therefore to find it, you uſe the *Lines of Tangents*, thus.

Take the lateral Diſtance 230 from the Line of Lines, and make it a parallel Diſtance on the Tangent Radius, viz. from 45° to 45°, then the parallel Tangent from 36° 30′, to 36° 30′, will meaſure laterally on the Line of Lines 170, 19, as before, for the Side A C.

EXAMPLE V. In the LINES OF SECANTS.

In the ſame Triangle, in the Baſe A B, and the Angles at B and C given, as before, to find the Side or Hypothenuſe B C. Here B C is the Secant of the Angle B.

Take the lateral Diſtance 230 on the Line of Lines, make it a parallel Diſtance at the Radius or Beginning of the *Lines of Secants*; then the parallel Secant of 60° 30′ will meaſure laterally on the Line of Lines 287, 12, for the Length of B C, as required.

EXAMPLE

EXAMPLE VI. In the LINES OF SINES AND TANGENTS conjointly.

In the Solution of Spherical Triangles, you uſe the *Line of Sines* and *Tangents* only, as in the following Example. In the Spherical Triangle A B C (Fig, 11.) right angled at A, there is given the Side A B = 36° 15′. and the adjacent Angle B = 42° 34′, to find the Side A C. The Analogy is Radius : Sine of A B : : Tangent of B : Tangent of A C; therefore make the lateral Sine of 36° 15′ a Parallel at Radius, or between 90 and 90; then the parallel Tangent of 42° 34′ will give the lateral Tangent of 28° 30′ for the Side A C.

EXAMPLE VII. In the LINES OF POLYGONS.

It has been obſerved that the Chord of 60 Degrees is equal to Radius; and 60° is the 6th Part of 360°; therefore ſuch a Chord is the Side of a *Hexagon*, inſcribed in a Circle. So that in the *Line of Polygons*, if you make the parallel Diſtance between 6 and 6, the Radius of a Circle, as A C (Fig. 12.) then if you take the parallel Diſtance between 5 and 5, and place it from A to B, the Line A B will be the Side of a *Pentagon* A B D E F, inſcribed in the Circle; in the ſame Manner may any other Polygon, from 4 to 12 Sides, be inſcribed in a Circle, or upon any given Line A B.

VIII. Of GUNTER'S LINES.

We have now ſhewn the Uſe of all that are properly called *Sectoral Lines*, or that are to be uſed *Sector-wiſe*: but there is another Set of Lines uſually put upon the Sector, that will in a more ready and ſimple Manner give the Anſwers to the Queſtions in the above Examples, and theſe are called *Artificial* LINES of NUMBERS, SINES, and TANGENTS; becauſe they are only the Logarithms of the natural Numbers, Sines and Tangents, laid upon Lines of Scales, which Method was firſt invented by Mr. *Edmund* GUNTER, and is the Reaſon why they have ever ſince been called *Gunter's Lines*, or the *Gunter*.

Logarithms are only the *Ratios of Numbers*, and the Ratios of all proportional Numbers are equal. Now all Queſtions in *Multiplication*, *Diviſion*, the *Rule of Three*, and the *Analogies* of *Plain and Spherical Trigonometry*, are all ſtated in proportional Numbers or Terms; therefore, if in the Compaſſes you take the Extent (or Ratio) between the firſt and ſecond Terms, that will always be equal to the Extent (or Ratio) between the 3d and 4th Terms; and conſequently, *if with the Extent between the 1ſt and 2d Terms, you place one Foot of the Compaſſes on the Third Term, then turning the Compaſſes about, the other Foot will fall on the Fourth Term ſought.*

Thus in Example I. of the three given Numbers 4, 7, and 16, if you take the Extent from 4 to 7 in the Compaſſes, and place one Foot in 16, the other will fall on 28 the Anſwer, in the Line of Numbers marked *n*.

Again, the *Artificial Line of Numbers and Sines*, are uſed together in Plain Trigonometry, as in Example III. where the two Angles B and C, and the Side A B are given, for here if you take the Extent of the two Angles 53° 30′ and 36° 30′ in the Line of Sines, mark'd *s*, then placing one Foot upon 230 in the Line of Numbers *n*, the other will reach to 170,19 the Anſwer.

Alſo the *Line of Numbers and Tangents* are uſed conjointly, as in the Example IV. for take in the Line of Tangents *t*, the Extent from 45° (Radius) to 36° 30′, that will reach from 230 to 170,19 the Anſwer as before.

Laſtly, the *Artificial Lines of Sines and Tangents* are uſed together in the Analogies of *Spherical Triangles*.

Thus Example VI. is ſolved by taking in the Line of Sines *s*, the Extent from 90° (Radius) to 36° 15′, then that in the Line of Tangents *t*, will reach from 42° 34′ to 28° 30′, the Anſwer required.

I ſhall only further obſerve that each Pair of Sectoral Lines contain the ſame Angle, viz. 6 Degrees in the common 6 Inch Sector; therefore to open theſe Lines to any given Angle, as 35° for Inſtance, you have only to take 35° laterally from the *Line of Chords*, and apply it *parallel-wiſe* from 60° to 60° in the ſame Lines, and they will all be opened to the given Angle of 35°.

If to the Angle 35° you add the Angle 6°, which they contain, the Sum is 41°; then take 41° laterally from the Line of Chords, and apply it parallel from 60 to 60, then will the Sides or Edges of the Sector contain the ſame Angle of 35 Degrees. And in this Caſe the Sector becomes a general *Recipe-Angle*, which is an Inſtrument for taking the Quantity of any Angle contained between two *inclining Planes*, as thoſe in *Fortifications*, &c.

IX. Of PROPORTIONAL COMPASSES.

Though this Sort of Compaſſes does not pertain to a Common Caſe of Inſtruments, yet a ſhort Account of their Nature and Uſe may not be unacceptable to thoſe who are not acquainted with them. They conſiſt of two Parts or Sides of Braſs, which lie upon each other, ſo nicely as to appear but one when they are ſhut. Theſe Sides eaſily open, and move about a Center, which is itſelf moveable in a hollow Canal cut through the greateſt Part of their Length. To this Center on each Side is affixed a Sliding Piece of a ſmall Length, with a fine Line drawn on it ſerving as an *Index*, to be ſet againſt other Lines or Diviſions placed upon the Compaſſes on both Sides. Theſe Lines are, (1.) A LINE of LINES, (2). A LINE of SUPERFICIES, AREAS or PLANS. (3.) A LINE of SOLIDS. (4.) A LINE of CIRCLES, or rather of *Polygons* to be inſcribed in Circles.

Theſe Lines are all unequally divided, the Three firſt from 1 to 10; the laſt from 6 to 20; their Uſes are as follows:

By the LINE of LINES you *divide a given Line into any Number of Equal Parts*; for by placing the Index againſt 1, and ſcrewing it faſt, if you open the Compaſſes, then the Diſtance between the Points at each End will be equal.—If you place the Index againſt 2, and open the Compaſſes, the Diſtance between the Points of the longer Legs will be twice the Diſtance between the ſhorter ones; and thus a Line is *biſected*, or divided into two equal Parts. If the Index be placed againſt 3, and the Compaſſes opened, the Diſtances between the Points will be as 3 to 1, and ſo a Line is divided into three equal Parts; and ſo you proceed for any other Number of Parts under 10.

The Number of the LINE OF PLANS anſwer to the *Squares* of thoſe in the Line of Lines; for becauſe *Superficies or Plans are to each other as the Squares of their Like Sides*, therefore if the Index be placed againſt 2 in the Line of Plans; then the Diſtance between the ſmall Points will be the Side of a Plan whoſe Area is 1; but the Diſtance of the larger Points will be the like Side of a Plan whoſe Area is 2, or twice as big.—If the Index be placed at 3, and the Compaſſes opened, the Diſtances between the Points at each End will be the *like Sides of Plans*, whoſe Areas are as 1 to 3, and ſo of others.

The Numbers of the LINE of SOLIDS anſwer to the *Cubes* of thoſe in the *Line of Lines*; becauſe *all Solids are to each other as the Cubes of their like Sides or Diameters*;

therefore, if the Index be placed to N° 2, 3, 4, &c. in the *Line of Solids*, the Diſtances between the leſſer and larger Points will be the like Sides of Solids, which are to each other as 1 to 2, 1 to 3, 1 to 4, &c. For Example, if the Index be placed at 10, and the Compaſſes be opened, ſo that the ſmall Points may take the Diameter of a Bullet weighing 1 Ounce, then the Diſtance between the larger Points will be the Diameter of a Bullet or Globe of 10 Ounces, or which is 10 Times as big.

Laſtly, the Numbers in the LINE OF CIRCLES are the *Sides of Polygons* to be inſcribed in a given Circle, or by which a Circle may be divided into thoſe Equal Parts from 6 to 20. Thus if the Index be placed at 6, the Points of the Compaſſes at either End, when opened to the Radius of a given Circle, will contain the Side of a *Hexagon*, or divide the Circle into 6 equal Parts.—If the Index be placed againſt 7, and the Compaſſes opened, ſo that the larger Points may take in the Radius of the Circle; then the ſhorter Points will divide the Circle into 7 equal Parts for inſcribing a *Heptagon*.——Again, placing the Index to 8, and opening the Compaſſes, the larger Points will contain the Radius, and the leſſer Points divide the Circle into 8 equal Parts, for inſcribing an *Octagon* or *Square*. And thus you proceed for others.

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